Key Concepts

- If two factors have a product of zero, then one or both of the factors must equal zero.
- A quadratic equation has degree two and has a single unknown. For example, $x^2 + 2x - 3 = 0$ is a quadratic equation.
- Some quadratic equations can be solved by factoring.
- To solve a quadratic equation by factoring, first write the equation in the form $ax^2 + bx + c = 0$, and then factor the left side. Next, set each factor equal to zero, and solve for the unknown.

  For example,

$$x^2 + 2x = 3$$
$$x^2 + 2x - 3 = 0$$
$$(x + 3)(x - 1) = 0$$
$$x + 3 = 0 \quad \text{or} \quad x - 1 = 0$$
$$x = -3 \quad \text{or} \quad x = 1$$

- The solutions to a quadratic equation are also known as the roots of the equation.

Communicate Your Understanding

C1 When you are factoring to solve a quadratic equation such as $x^2 + 2x - 3 = 0$, the right side needs to be equal to zero. Why?

C2 Describe how you would solve each quadratic equation by factoring.

a) $3x^2 + 12x + 9 = 0$

b) $2x^2 - 11x = -15$

Practise

For help with questions 1 to 5, see Example 1.

1. Solve.

a) $(x + 5)(x + 2) = 0$

b) $(x - 3)(x + 4) = 0$

c) $(x - 1)(x - 7) = 0$

d) $x(x + 9) = 0$

e) $(2x + 3)(x - 5) = 0$

f) $(2x - 1)(3x + 4) = 0$

g) $(3x - 5)(4x - 3) = 0$

2. Solve and check.

a) $x^2 + 8x + 12 = 0$

b) $h^2 + 9h + 18 = 0$

c) $m^2 + 3m = 0$

d) $w^2 - 18w + 56 = 0$

e) $x^2 - 2x = 0$

f) $c^2 - 17c + 30 = 0$

g) $n^2 + 9n - 22 = 0$

h) $y^2 - 11y = 0$

3. Solve.

a) $3x^2 + 28x + 9 = 0$

b) $4k^2 + 19k + 15 = 0$

c) $8y^2 - 22y + 15 = 0$

d) $16b^2 - 1 = 0$

e) $10m^2 + 30m = 0$

f) $4x^2 - 12x + 9 = 0$

4. Solve.

a) $x^2 + 5x = -4$

b) $8c + 15 = -c^2$

c) $k^2 = 13k - 12$

d) $b^2 + 1 = -2b$

e) $m^2 = 300 - 20m$

f) $y^2 = 7y$

5. Solve.

a) $2m^2 = -7m - 6$

b) $9x^2 = x + 8$

c) $4y^2 - 12y = -9$

d) $-5 = 2p - 16p^2$

e) $12m^2 = 10 - 37m$

f) $3w^2 + 22w = -7$

For help with questions 6 and 7, see Example 2.

6. Solve.

a) $-x^2 - 10x - 16 = 0$

b) $3t^2 + 24t + 45 = 0$

c) $6d^2 + 15d = -9$

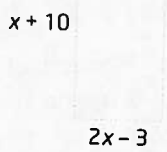
d) $-10g^2 + 32g = 6$

Connect and Apply

7. A basketball is tossed from the top of a 3-m wall. The path of the basketball is defined by the relation $y = -x^2 + 2x + 3$, where x represents the horizontal distance travelled, in metres, and y represents the height, in metres, above the ground. How far has the basketball travelled horizontally when it lands on the ground?

For help with question 8, see Example 3.

8. A rectangle has dimensions $x + 10$ and $2x - 3$. Determine the value of x that gives an area of 54 cm^2.



9. Write a quadratic equation in factored form for each situation.

a) The roots of the equation are 5 and 4.

b) The roots of the equation are -2 and 3.

10. a) Write a quadratic equation in the form $ax^2 + bx + c = 0$ with roots of 6 and -7.

b) What would happen to the roots if you multiplied both sides of the equation in part a) by 3? Explain.

11. Write a quadratic equation with roots of $\frac{2}{3}$ and $-\frac{4}{5}$ in the form $ax^2 + bx + c = 0$, where a, b, and c are integers.

12. a) Create an example of a quadratic equation that can be factored and solved with integer solutions.

b) Create an example of a quadratic equation that can be factored and solved with non-integer solutions.

13. Create an example of a quadratic equation that cannot be solved by factoring. Explain why it cannot be factored.

14. The hypotenuse of a right triangle measures 29 cm. One leg is 1 cm shorter than the other. What are the lengths of the legs?